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Cherenkov Radiation in Moving Medium

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Abstract Cherenkov radiation in uniformly moving homogeneous isotropic medium without dispersion is studied. Formula for the spectrum of Cherenkov radiation of fermion was derived for the case when the speed of the medium is less than the speed of light in this medium at rest. The properties of Cherenkov spectrum are investigated.

Keywords Cherenkov radiation · Moving medium · Spectrum

1 Introduction

Cherenkov radiation is an outstanding phenomenon, which has applications in a range of various domains. Classical description of this phenomenon was obtained a long time ago. Also there exist a number of quantum considerations of this effect. Spectrum of Cherenkov radiation in the medium at rest was calculated in a number of papers (see, for example [10–12]). There are not so many articles and reviews regarding Cherenkov radiation in moving medium (see, for example [13]). However, in [13] only classical radiation without quantum corrections was considered.

The aim of this article is to obtain the formula of Cherenkov radiation spectrum in moving medium by systematic calculation in the framework of quantum in-medium QED. Also we are going to analyze how the shape of this spectrum and angle distribution of this radiation depend on the parameters of the considered system.

Calculation of the spectrum of Cherenkov radiation in moving medium is of some interest because of its relation to phenomena in nucleus-nucleus collisions. Significant experimental data concerning an existence of double-humped structure of away-side azimuthal correlations got in the experiments STAR [1, 2] and PHENIX [3–5] at RHIC aroused a new wave

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of interest to a phenomenon of Cherenkov radiation of gluons first considered in [6–8]. In the relation with this phenomenon it is interesting to analyze Cherenkov radiation in moving medium because the medium that is formed in nucleus-nucleus collisions is expanding. It is important that in this situation the radiating particle and the medium move in the same direction.

2 Effective Notion of Medium in QED

Let us consider the effect of medium for processes in QED. We will use the simplest model with constant dielectric permittivity and neglect temporal and spatial dispersion. Then the dielectric permittivity tensor [9] in the reference frame, in which the medium is at rest takes the form

$$\varepsilon^{\rho\sigma} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

In arbitrary reference frame the dielectric permittivity tensor can be written as follows

$$\varepsilon_{\mu\nu} = g_{\mu\nu} + (n^2-1)\widetilde{v}_\mu \widetilde{v}_\nu, \tag{2.1}$$

where $n=\sqrt{\varepsilon}$ is a refraction index, and $\widetilde{v}^\mu = (\frac{1}{\sqrt{1-v^2}}, \frac{\overrightarrow{v}}{\sqrt{1-v^2}})$ is a 4-velocity of the medium and $\overrightarrow{v}$ is a spatial velocity.

The aim of the present article is a calculation of the spectrum of Cherenkov radiation in moving medium. The calculation will be made in the Lorentz gauge $\varepsilon_{\mu\nu}\partial^\mu A^\nu = 0$. The first thing to find out is the photon dispersion law in moving medium. Equation of motion of the field A^μ in the Lorentz gauge takes the form

$$\varepsilon_{\mu\rho}\varepsilon_{\nu\sigma}\partial^\mu\partial^\rho A^\sigma(x) = 0. \tag{2.2}$$

In momentum representation one has, correspondingly

$$\varepsilon_{\mu\rho}\varepsilon_{\nu\sigma} q^\mu q^\rho A^\sigma(q) = 0, \tag{2.3}$$

so that the covariant dispersion law of the photon in medium reads

$$\varepsilon_{\mu\rho} q^\mu q^\rho = 0. \tag{2.4}$$

Substituting the definition of the dielectric permittivity tensor from (2.1) we have

$$q^2 + (n^2-1)(q\widetilde{v})^2 = 0, \tag{2.5}$$

$$\left(1+\frac{n^2-1}{1-v^2}\right)(q^0)^2 - 2q^0\frac{n^2-1}{1-v^2}|\mathbf{q}|v\cos\theta - |\mathbf{q}|^2\left(1-\frac{n^2-1}{1-v^2}v^2\cos^2\theta\right) = 0, \tag{2.6}$$

where θ is the angle between the photon momentum $\mathbf{q}$ and the velocity of the medium $\mathbf{v}$. The quadratic equation (2.6) has two solutions

$$q^0 = \frac{\frac{n^2-1}{1-v^2}v\cos\theta \pm (1+\frac{n^2-1}{1-v^2}(1-v^2\cos^2\theta))^{\frac{1}{2}}}{1+\frac{n^2-1}{1-v^2}}|\mathbf{q}|.$$

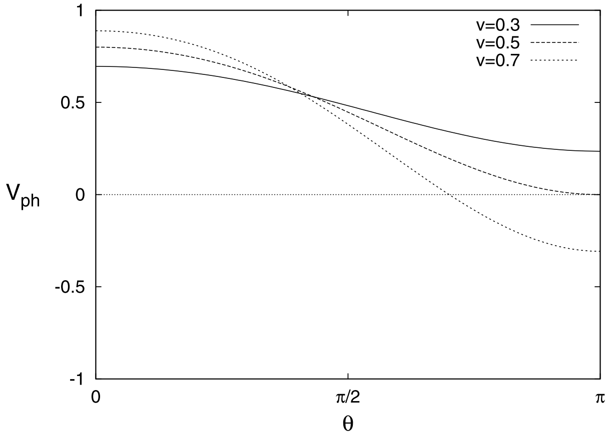


Fig. 1 Phase velocity of the photon in medium for $n = 2.0$. *Solid line* $v = 0.3$, *dashed line* $v = 0.5$, *dotted line* $v = 0.7$

The solution with the sign "−" in the limit $v = 0$ transforms to $q^0 = -\frac{|\mathbf{q}|}{n}$, the solution with the sign "+" in the limit $v = 0$ transforms to $q^0 = \frac{|\mathbf{q}|}{n}$. That is why we choose the solution with the sign "+" as physically sensible. In our case phase velocity $V_{ph}(\mathbf{q})$ determined by the dispersion relation $q^0 = V_{ph}|\mathbf{q}|$ depends on the direction of the photon propagation

$$V_{ph} = \frac{\frac{n^2-1}{1-v^2} v \cos\theta + (1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta))^{\frac{1}{2}}}{1 + \frac{n^2-1}{1-v^2}}. \tag{2.7}$$

Such a photon dispersion law in medium was derived in [13].

It is easy to verify that if $|v| < \frac{1}{n}$, the phase velocity is positive for all θ, but if $|v| \geqslant \frac{1}{n}$, there exists the region of θ, where $V_{ph} < 0$. This statement is illustrated by Fig. 1. Below we examine the case, when the velocity of the medium is smaller, than the phase speed of light in this medium at rest. Conjointly, one can easily prove that $V_{ph} < 1$ for all v satisfying $|v| < 1$.

In what follows we shall need an expression for the photon propagator $D_{\mu\nu}(q)$ in the Lorentz gauge. According to Ter-Mikaelyan [9] it is represented by the formula

$$D_{\mu\nu}(q) = \frac{-i(g_{\mu\nu} + (n^{-2} - 1)\widetilde{v}_\mu \widetilde{v}_\nu)}{q^2 + (n^2 - 1)(q\widetilde{v})^2 + i\varepsilon}. \tag{2.8}$$

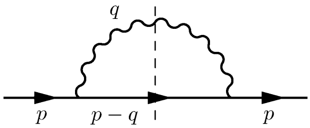
3 Spectrum of Cherenkov Radiation

The spectrum of Cherenkov radiation reads (see, for example, [12])

$$P(\omega) = \frac{\omega}{E} e^2 \int \frac{d^4q}{(2\pi)^4} \delta\left(\omega - V_{ph}(\mathbf{q})|\mathbf{q}|\right) \mathrm{Im}[i\mathcal{M}(p \to p)], \tag{3.1}$$

where ω is the photon energy, E is the energy of the radiating particle, p – its 4-momentum, $\mathcal{M}(p \to p)$ is the matrix element, the δ-function originates from a unit frequency projection operator and $V_{ph}(\mathbf{q})$ is phase speed of a photon with momentum $|\mathbf{q}|$. The δ-function in (3.1) ensures that the radiated photon is on the mass shell. The spectrum $P(\omega)$ corresponds to probability distribution of energy radiated in the interval of photon energies $[\omega, \omega + d\omega]$ per unit time.

Leading order contribution to the imaginary part of the matrix element $\mathcal{M}(p \to p)$ is given by the cut of self-energy diagram.



It is important, that if we considered such a diagram in vacuum, putting initial and final fermion on the mass-shell, this matrix element would not give any contribution to the imaginary part. The decay of fermion into fermion and photon is possible only in the medium.

The expression for the matrix element with photon and electron propagators

$$i\mathcal{M}(p \to p) = \int \frac{d^4q}{(2\pi)^4}\overline{u}(p)(-ie\gamma^{\mu})S(p-q)(-ie\gamma^{\nu})u(p)D_{\mu\nu}(q). \tag{3.2}$$

The expression for the spectrum of Cherenkov radiation in medium at rest was calculated, for example, in articles [10, 11]. Cherenkov radiation spectrum was also derived in approximation of soft photons in article [12]. It reads

$$P(\omega|v=0,\beta) = \alpha\omega\beta\left\{1 - \frac{1}{n^2\beta^2}\left(1 + \frac{\omega}{2E}(n^2-1)\right)^2 + \frac{\omega^2}{2E^2}\frac{n^2-1}{\beta^2}\right\}, \tag{3.3}$$

where β is the speed of the fermion. For the angle of Cherenkov radiation of the photon with energy ω one has

$$\cos\theta_e = \frac{1}{n\beta}\left(1 + \frac{n^2-1}{2}\frac{\omega}{E}\right). \tag{3.4}$$

The condition $|\cos\theta_e| \leq 1$ limits the energy of Cherenkov photons. The following investigation of the properties of Cherenkov radiation in moving medium also contain the conclusions concerning the case when the medium is at rest.

Now we return to the formula (3.1) for Cherenkov spectrum. Then we substitute the propagator of the photon and the propagator of the fermion

$$S(p-q) = \frac{i(\not p - \not q + m)}{(p-q)^2 - m^2 + i\varepsilon},$$

in (3.2), where $\widetilde{v}_{\mu}$ – 4-velocity of the medium. The propagator of fermion does not contain any information about the medium, because only photon field is affected by the medium. Formula (3.1) transforms to

$$P(\omega) = \frac{\omega}{E}e^2\int\frac{d^4q}{(2\pi)^4}\delta\left(\omega - V_{ph}|\mathbf{q}|\right) \times \operatorname{Im}\frac{i\overline{u}(p)\gamma^{\mu}(\not p - \not q + m)\gamma^{\nu}u(p)(g_{\mu\nu} + (n^{-2}-1)\widetilde{v}_{\mu}\widetilde{v}_{\nu})}{(q^2 + (n^2-1)(q\widetilde{v})^2 + i\varepsilon)((p-q)^2 - m^2 + i\varepsilon)}. \tag{3.5}$$

There we must say that in this article the case, when the velocity of the medium and the momentum of the particle are co-directed, is considered for simplicity.

Handling Dirac algebra and denoting the numerator we have

$$N(p,q,\widetilde{v}) = -4E^2\beta^2\left[1-\frac{1}{\beta^2}+\left(1-\frac{1}{n^2}\right)\frac{(1-\beta v)^2}{\beta^2(1-v^2)}\right.$$
$$-\frac{q^0}{2\beta^2 E}\left\{1+\frac{1}{n^2}+2\left(1-\frac{1}{n^2}\right)\frac{1-\beta v}{1-v^2}\right\}$$
$$\left.+\frac{|\mathbf{q}|}{2\beta E}\left\{1+\frac{1}{n^2}+2\left(1-\frac{1}{n^2}\right)\frac{v(1-\beta v)}{\beta(1-v^2)}\right\}\cos\theta\right], \tag{3.6}$$

where θ is the angle between the velocity of the medium and the momentum of the photon $|\mathbf{q}|$.

After integrating (3.5) over $|\mathbf{q}|$ we obtain

$$P(\omega) = \frac{\omega^3}{E(1+\frac{n^2-1}{1-v^2})}e^2\frac{1}{(2\pi)^3}\int dq^0\int_{-1}^{1} d(\cos\theta)\frac{1}{V_{ph}^3}N(p,q,\widetilde{v})|_{|\mathbf{q}|=\omega/V_{ph}}$$
$$\times \operatorname{Im}\frac{i}{(q^0-q^0_{1+})(q^0-q^0_{1-})(q^0-q^0_{2+})(q^0-q^0_{2-})}.$$

There are four poles in this expression: two in the upper half plane and two in the lower one (see (A.1), (A.3), (A.2)).

To perform the integration over q^0 we close the integration contour in the lower half plane and find the residues

$$P(\omega) = \frac{\omega^3}{E(1+\frac{n^2-1}{1-v^2})}e^2\frac{1}{(2\pi)^3}\int_{-1}^{1} d(\cos\theta)\frac{1}{V_{ph}^3}$$
$$\times 2\pi \operatorname{Im}\left[\frac{\overline{u}Nu|_{q^0=q^0_{1+}}}{D_{1\varepsilon}}+\frac{\overline{u}Nu|_{q^0=q^0_{2+}}}{D_{2\varepsilon}}\right], \tag{3.7}$$

$$D_{1\varepsilon} = (q^0_{1+}-q^0_{1-})(q^0_{1+}-q^0_{2+})(q^0_{1+}-q^0_{2-}), \tag{3.8}$$
$$D_{2\varepsilon} = (q^0_{2+}-q^0_{1+})(q^0_{2+}-q^0_{1-})(q^0_{2+}-q^0_{2-}). \tag{3.9}$$

It was proved in Appendix B that $D_{1\varepsilon}$ does not contribute to the integral over $\cos\theta$. This means that only one pole $q_0 = q_{2+}$ contributes to the final expression. Let's denote the Cherenkov angle θ_e and introduce convenient notations

$$x = \frac{\omega}{E}, \qquad y = \frac{n^2-1}{1-v^2}. \tag{3.10}$$

Then

$$\operatorname{Im}\frac{1}{D_{2\varepsilon}} = -\frac{\pi}{2\omega E^2(1-B(\theta_e))\beta\frac{x}{V_{ph}^3}\frac{V_{ph}+\frac{xv}{\beta}\sqrt{y(1-V_{ph}^2)}}{V_{ph}+\sqrt{y(1-V_{ph}^2)}}}\delta(\cos\theta-\cos\theta_e). \tag{3.11}$$

The formula (3.7) can thus be rewritten as

$$P(\omega) = \frac{\omega^3}{E(1+\frac{n^2-1}{1-v^2})}e^2\frac{1}{(2\pi)^2}\int_{-1}^{1} d(\cos\theta)\frac{1}{V_{ph}^3}\operatorname{Im}\left[\frac{\overline{u}Nu|_{q^0=\omega}}{D_{2\varepsilon}}\right]. \tag{3.12}$$

We substitute the obtained expression (3.11) in the expression (3.12) for Cherenkov radiation spectrum and integrate by $\cos\theta$. Expression for Cherenkov angle $\cos\theta_e$ (C.8) was obtained in Appendix C. After lengthy calculations we get the final answer for the spectrum

$$\begin{aligned}
P(\omega|v>0,\beta) &= \frac{\alpha\omega\beta}{1+\sqrt{2(1+\frac{n^2-1}{1-v^2}\frac{\omega}{E}\frac{v}{\beta}(1-\frac{v}{\beta})-\sqrt{1+2\frac{n^2-1}{1-v^2}\frac{\omega}{E}\frac{v}{\beta}(1-\frac{v}{\beta})})}} \\
&\quad \times\left[1-\frac{1}{\beta^2}+\left(1-\frac{1}{n^2}\right)\frac{(1-\beta v)^2}{\beta^2(1-v^2)}\right. \\
&\quad -\frac{\omega}{2\beta^2 E}\left\{1+\frac{1}{n^2}+2\left(1-\frac{1}{n^2}\right)\frac{1-\beta v}{1-v^2}\right\} \\
&\quad +\frac{1}{2}\left\{1+\frac{1}{n^2}+2\left(1-\frac{1}{n^2}\right)\frac{v(1-\beta v)}{\beta(1-v^2)}\right\}\frac{1-v^2}{v^2(n^2-1)} \\
&\quad \times\left(1+\frac{n^2-1}{1-v^2}\frac{\omega}{E}\frac{v}{\beta}\left(1-\frac{v}{\beta}\right)\right. \\
&\quad \left.\left.-\sqrt{1+2\frac{n^2-1}{1-v^2}\frac{\omega}{E}\frac{v}{\beta}\left(1-\frac{v}{\beta}\right)}\right)\right].
\end{aligned} \tag{3.13}$$

For the Cherenkov angle we get (see Appendix C)

$$\cos\theta_e = \frac{1+\frac{\beta}{xyv}(1-\sqrt{1+2xy\frac{v}{\beta}(1-\frac{v}{\beta})})}{\sqrt{v^2+\frac{2\beta^2}{x^2}(\frac{xv}{\beta}(1-\frac{v}{\beta})+\frac{1}{y}(1-\sqrt{1+2xy\frac{v}{\beta}(1-\frac{v}{\beta})}))}}. \tag{3.14}$$

The condition $|\cos\theta_e|\leq 1$ provides the upper cutoff for the spectrum.

Let us start with analysis of the dependence of Cherenkov angle on the energy of the emitted photon. This dependence (3.14) is illustrated on Figs. 4 and 5.

Then we examine only the spectra for positive v and pay special attention to the ultra relativistic case ($\beta = 1$). These spectra are pictured on Figs. 2 and 3. One can see the hardening of the spectrum for $\beta = 1$ with the increment of medium speed v. This hardening is explained below.

There is an important thing to notice about the relationships on Figs. 4 and 5. All momenta of radiated Cherenkov photons lie inside the cone with the opening angle $2\theta_e^{\max}$, corresponding to $x = 0$ (Cherenkov photon with zero energy)

$$\cos\theta_e^{\max} = \frac{1}{\beta\sqrt{1+\frac{n^2-1}{1-v^2}(1-\frac{v}{\beta})^2}}. \tag{3.15}$$

The dependence of this angle on the speed of the medium is illustrated on Fig. 6. It can be easily verified analytically that the function (3.15) is an increasing function of v. Therefore the opening angle of the Cherenkov cone decreases as the speed of medium increases.

One can see from Figs. 4 and 5 that with increasing photon energy the angle between the photon momentum and the axis of the cone becomes smaller. Thus the photon with the maximal energy is radiated with the momentum aligned at the direction of motion of the fermion. Therefore this energy is the upper bound of the Cherenkov spectrum. This

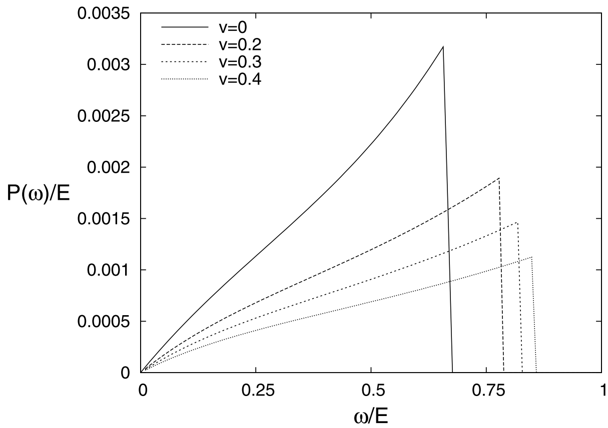


Fig. 2 Spectrum of Cherenkov radiation in moving medium. $n = 2.0$, $\beta = 1.0$. *Solid line* $v = 0$, *thick dashed line* $v = 0.2$, *thin dashed line* $v = 0.3$, *dotted line* $v = 0.4$

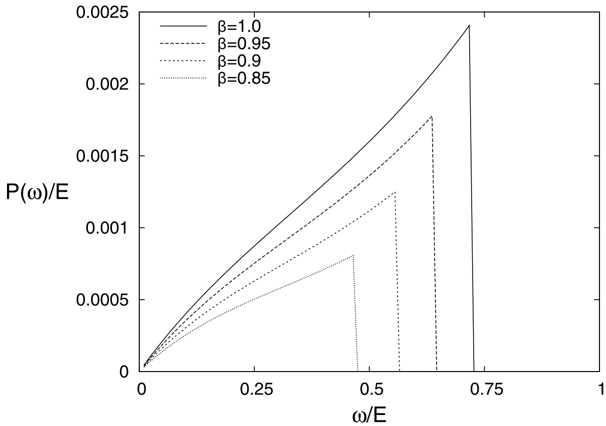


Fig. 3 Spectrum of Cherenkov radiation in moving medium. $n = 2.0$, $v = 0.1$. *Solid line* $\beta = 1.0$, *thick dashed line* $\beta = 0.95$, *thin dashed line* $\beta = 0.9$, *dotted line* $\beta = 0.85$

statement is easily derived analytically because the function $\cos\theta_e(x)$ is monotonous. Using the energy-momentum conservation law, we find this energy (in units of the fermion energy)

$$x_{\max} = \frac{2V_{ph}(\theta)(\beta\cos\theta - V_{ph}(\theta))}{1 - V_{ph}^2(\theta)}\bigg|_{\theta=0} = \frac{2(1 - \beta\frac{n+v}{1+nv})}{1 - (\frac{n+v}{1+nv})^2}. \tag{3.16}$$

This function is a decreasing function of β for all v, $|v| < \frac{1}{n}$. We see from Fig. 3 that the upper bound of the spectrum decreases as the velocity of the fermion increases. The dependence of the upper bound of the spectrum on the velocity of the medium is illustrated on Fig. 7.

From Fig. 7 one can see that in the ultra-relativistic case ($\beta = 1$) the relationship is monotonous, i.e. the maximal energy of Cherenkov photon increases with growing v. For $\beta = 1$ one has

$$x_{\max} = 2n - \frac{2(n-1)}{(n+1)(1+v)}.$$

This function is an increasing function of v.

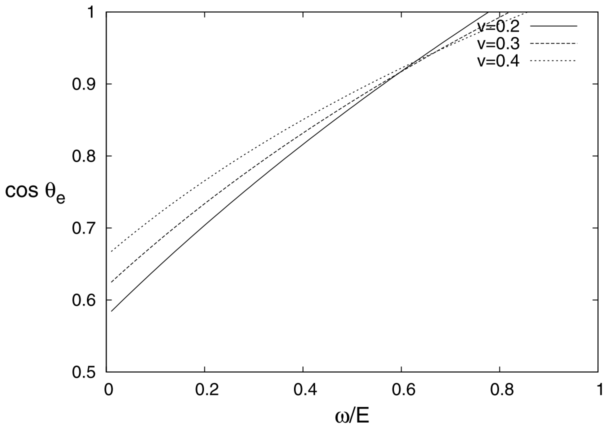


Fig. 4 Dependence of Cherenkov angle on the energy of emitted photon. $n = 2.0$, $\beta = 1.0$. *Solid line* $v = 0.2$, *dashed line* $v = 0.3$, *dotted line* $v = 0.4$

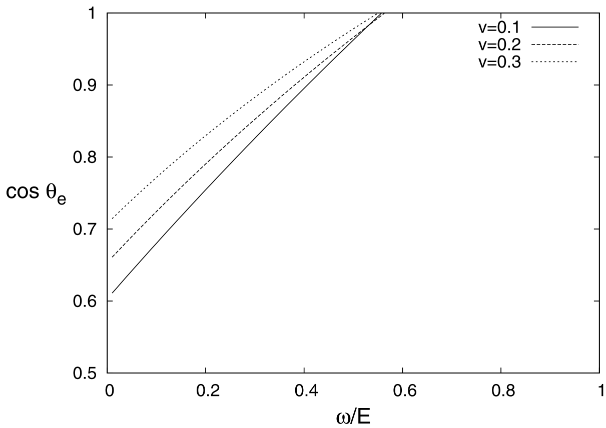


Fig. 5 Dependence of Cherenkov angle on the energy of emitted photon. $n = 2.0$, $\beta = 0.9$. *Solid line* $v = 0.1$, *dashed line* $v = 0.2$, *dotted line* $v = 0.3$

Now we can analyze the geometry of the radiation spectrum. Let $P(\theta_e)$ be energy per unit time radiated in the solid angle $d\Omega_e = \sin\theta_e d\theta_e d\varphi$, where θ_e is Cherenkov angle and φ is the polar angle relative to the axis of fermion's motion. Then it is related with $P(\omega)$ in the following way

$$P(\theta_e) = \frac{P(\omega(\theta_e))}{2\pi} \frac{\partial \omega(\theta_e)}{\partial \cos\theta_e}, \tag{3.17}$$

$$\omega(\cos\theta_e) = \frac{2V_{ph}(\theta)(\beta\cos\theta_e - V_{ph}(\theta_e))}{1 - V_{ph}^2(\theta_e)}. \tag{3.18}$$

This spectra are pictured on Figs. 8 and 9. At first, we notice that the opening angle of Cherenkov cone decreases as the speed of the medium increases, which confirms our conclusion. We see that the radiation is most intensive along the fermion's propagation axis.

Then naturally arises a question why the presented picture of Cherenkov radiation differs from standard two-bumped structure. Cherenkov observed a two-bumped structure, but we don't see this structure in the presented picture of Cherenkov radiation. The origin of this contradiction is our model of medium. In Sect. 2 we assumed that the medium doesn't have frequency dispersion and the dielectric permittivity remains constant for all photon energies

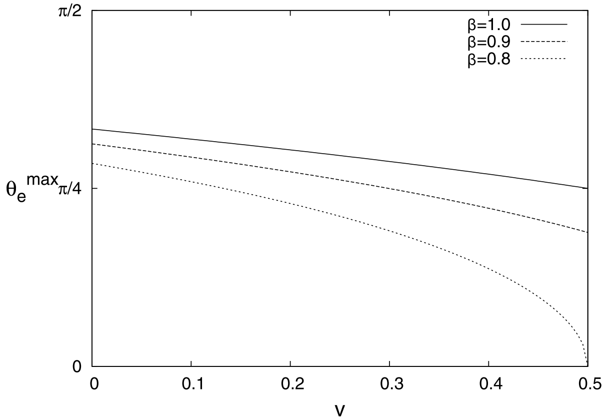


Fig. 6 Opening angle of the Cherenkov cone. $n = 2.0$. *Solid line* $\beta = 1.0$, *dashed line* $\beta = 0.9$, *dotted line* $\beta = 0.8$

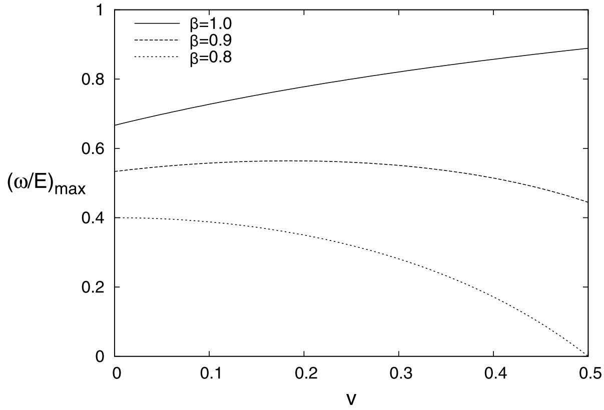


Fig. 7 The upper bound of the spectrum. $n = 2.0$. *Solid line* $\beta = 1.0$, *dashed line* $\beta = 0.9$, *dotted line* $\beta = 0.8$

ω. But in real situation this is not true, because the condition $n(\omega) > 1$ can be true only for restricted range of frequencies. Thus possible explanation of this contradiction is that the energy of radiating particles is much greater than the energies from this range. This means that the particle radiates only the photon whose energy is much less than the energy of the particle. Thus according to the previous reasonings the momenta of these Cherenkov photons are close to the surface of the Cherenkov cone. So we obtain the usual picture of Cherenkov radiation: Cherenkov photons are radiated close to the surface of Cherenkov cone.

One can examine a lot of interesting cases in moving media. For example, the situation when speed of medium is greater than phase speed of light in this medium at rest. It is possible to study this phenomenon using various relations between velocity of medium and velocity of a particle. In this article the simplest case was examined.

4 Conclusions

In this article was conducted the calculation of Cherenkov radiation spectrum in moving medium with usage of quantum field theory methods. There were also examined the dependence of this spectrum on the speed of the medium and the spatial distribution of the radi-

Fig. 8 Angle distribution of Cherenkov Radiation. $n = 2.0$, $\beta = 1.0$. *Solid line* $v = 0$, *thick dashed line* $v = 0.2$, *thin dashed line* $v = 0.3$, *dotted line* $v = 0.4$

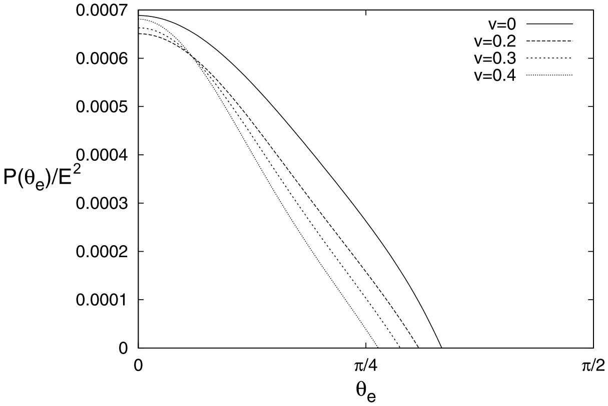
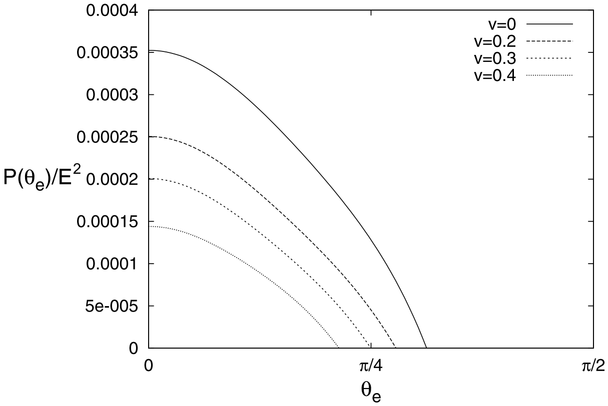


Fig. 9 Angle distribution of Cherenkov Radiation. $n = 2.0$, $\beta = 0.9$. *Solid line* $v = 0$, *thick dashed line* $v = 0.2$, *thin dashed line* $v = 0.3$, *dotted line* $v = 0.4$



ation. Spectrum of Cherenkov radiation in moving media differs from the spectrum when medium is at rest.

At first, the dispersion law of the photon is modified in moving medium. It reveals different behavior when the speed of medium is greater or lower than phase speed of light in medium at rest. When the speed of medium is greater than phase speed of light in medium at rest there is a region of directions where phase speed in negative.

Let us now summarize the obtained properties of the spectrum. In the case when the particle and the medium move in the same directions the spectrum for moving medium lies under the spectrum for medium at rest. Greater the speed of the medium, lower it lies. But the cut of the spectrum increases as the speed of the medium increases, and decreases as the speed of the fermion increases.

The geometrical structure of the Cherenkov radiation of moving medium has a particular property. All Cherenkov photons are radiated in the cone with the opening angle $2\theta_e^{\max}$ (where $\theta_e^{\max}$ is given by the formula (3.15)), whose axis coincides with the vector of fermion's momentum. This angle decreases as the speed of the medium increases. Then the photons with the greater energy have momenta that are closer to the axis of the cone.

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Appendix A: Pole Structure

Let us turn to the pole structure of the expression (3.5). We write down the denominator

$$(q^2+(n^2-1)(q\widetilde{v})^2+i\varepsilon)((p-q)^2-m^2+i\varepsilon).$$

Let's analyze the expression in the first brackets

$$(p-q)^2-m^2+i\varepsilon=p^2-2pq+q^2-m^2+i\varepsilon=q^2-2pq+i\varepsilon,$$
$$(q^0)^2-2Eq^0-|\mathbf{q}|^2+2\beta E|\mathbf{q}|\cos\theta=0.$$

This equation has two solutions. Taking into account, that because of δ-function $|\mathbf{q}|=\frac{\omega}{V_{ph}}$, we have

$$q^0_{1\pm}=E\left\{1\pm\sqrt{1+\frac{\omega^2}{E^2V_{ph}^2}-2\beta\frac{\omega}{EV_{ph}}\cos\theta}\right\}\mp i\varepsilon. \tag{A.1}$$

Now we handle the other two poles. We know the solutions of this expression, so we just write down two residuary poles

$$q^0_{2+}=\omega-i\varepsilon \tag{A.2}$$

$$q^0_{2-}=\frac{\frac{n^2-1}{1-v^2}v\cos\theta-(1+\frac{n^2-1}{1-v^2}(1-v^2\cos^2\theta))^{\frac{1}{2}}}{\frac{n^2-1}{1-v^2}v\cos\theta+(1+\frac{n^2-1}{1-v^2}(1-v^2\cos^2\theta))^{\frac{1}{2}}}\omega+i\varepsilon=B(\theta)\omega+i\varepsilon. \tag{A.3}$$

Then we must prove two lemmas regarding this poles. But at first we introduce the designation

$$A(\theta)=\sqrt{1+\frac{\omega^2}{E^2V_{ph}^2}-2\beta\frac{\omega}{EV_{ph}}\cos\theta}. \tag{A.4}$$

Lemma 1 *It is right for all θ that*

$$A(\theta)\geqslant\sqrt{1-\beta^2}>0. \tag{A.5}$$

Proof

$$A(\theta)=\sqrt{1-\beta^2\cos^2\theta+\left(\frac{\omega}{EV_{ph}}-\beta\cos\theta\right)^2}$$
$$\geqslant\sqrt{1-\beta^2\cos^2\theta}\geqslant\sqrt{1-\beta^2}>0. \qquad \square$$

Lemma 2 *For all θ it is right that*

$$E(1+A(\theta))>\omega. \tag{A.6}$$

Proof Let's assume the contrary. This means that there exists θ_0 that

$$\omega\geqslant E(1+A(\theta_0)),$$
$$\frac{\omega}{E}\geqslant 1+A(\theta_0).$$

Introducing the designation $x = \frac{\omega}{E}$, we have

$$\begin{aligned}
x - 1 &\geqslant A(\theta_0) > 0,\\
x^2 - 2x + 1 &\geqslant 1 + \frac{x^2}{V_{ph}^2} - 2\beta \frac{x}{V_{ph}} \cos\theta_0,\\
\cos\theta_0 &\geqslant \frac{V_{ph}}{\beta}\left(1 + \left(\frac{1}{V_{ph}^2} - 1\right)\frac{x}{2}\right) > \frac{V_{ph}}{\beta}\left(1 + \left(\frac{1}{V_{ph}^2} - 1\right)\frac{1}{2}\right)\\
&= \frac{V_{ph}^2 + 1}{2\beta V_{ph}} > \frac{1}{\beta} > 1.
\end{aligned}$$

It means that our assumption is not correct and the initial statement is correct

$$E(1 + A(\theta)) > \omega. \qquad \square$$

From the proved lemmas we can make a conclusion that the poles in the lower half plane never coincide.

Appendix B: Evaluation of the Integral

Let's examine the expressions $D_{1\varepsilon}$ and $D_{2\varepsilon}$ more carefully. We start from $D_{1\varepsilon}$

$$\begin{aligned}
D_{1\varepsilon} &= (q_{1+}^0 - q_{1-}^0)(q_{1+}^0 - q_{2+}^0)(q_{1+}^0 - q_{2-}^0)\\
&= 2(EA(\theta) - i\varepsilon)(E(1 + A(\theta)) - B(\theta)\omega - 2i\varepsilon)(E(1 + A(\theta)) - \omega),\\
\lim_{\varepsilon\to 0} D_{1\varepsilon} &= 2EA(\theta)(E(1 + A(\theta)) - B(\theta)\omega)(E(1 + A(\theta)) - \omega).
\end{aligned}$$

According to Lemma 1 function $A(\theta)$ is positive for all θ, and according to Lemma 2 $E(1 + A(\theta)) > \omega$, then these expressions equal 0 nowhere. Let's write down the function $B(\theta)$

$$B(\theta) = \frac{\frac{n^2-1}{1-v^2} v\cos\theta - (1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta))^{\frac{1}{2}}}{\frac{n^2-1}{1-v^2} v\cos\theta + (1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta))^{\frac{1}{2}}}.$$

We will prove that for $|v| < \frac{1}{n}$ the numerator of this expression is negative.

$$\begin{aligned}
&\frac{n^2-1}{1-v^2} v\cos\theta - \left(1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta)\right)^{\frac{1}{2}} < 0,\\
&\frac{n^2-1}{1-v^2} v\cos\theta < \left(1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta)\right)^{\frac{1}{2}},\\
&\frac{(n^2-1)^2}{(1-v^2)^2} v^2\cos^2\theta < 1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta),\\
&\frac{n^2-1}{1-v^2} v^2\cos^2\theta < 1.
\end{aligned}$$

It is easy to prove that this inequality is valid for all θ if $|v| < \frac{1}{n}$. But this in its part means that the stating $D_{1\varepsilon}$ does not equal 0 for any θ. Consequently, $D_{1\varepsilon}$ does not make any contribution to the expression with an imaginary part.

Appendix C: Calculation of Cherenkov Angle

Let's turn to $D_{2\varepsilon}$.

$$\begin{aligned} D_{2\varepsilon} &= (q^0_{2+} - q^0_{1+})(q^0_{2+} - q^0_{1-})(q^0_{2+} - q^0_{2-}) \\ &= ((\omega - E)^2 - E^2 A^2(\theta) + i\varepsilon)(\omega(1 - B(\theta)) - 2i\varepsilon), \\ \lim_{\varepsilon\to 0} D_{2\varepsilon} &= ((\omega - E)^2 - E^2 A^2(\theta))\omega(1 - B(\theta)). \end{aligned}$$

Expression in the second brackets doesn't equal 0 because $B(\theta) < 0$ for all θ. And from the stating in the first brackets we can find the Cherenkov angle. We write down the equation

$$(\omega - E)^2 - E^2 A^2(\theta) = 0. \tag{C.1}$$

Recalling the equality (A.4), we obtain the equation

$$\left(\frac{\omega}{E} - 1\right)^2 - \left(1 + \frac{\omega^2}{E^2 V_{ph}^2} - 2\beta\frac{\omega}{E V_{ph}}\cos\theta\right) = 0. \tag{C.2}$$

We rewrite (C.2)

$$(x - 1)^2 - \left(1 + \frac{x^2}{V_{ph}^2} - 2\beta\frac{x}{V_{ph}}\cos\theta\right) = 0. \tag{C.3}$$

We will solve this equation in the following way. At first we transform (C.3)

$$\cos\theta = \frac{1}{\beta V_{ph}}\left(V_{ph}^2 + (1 - V_{ph}^2)\frac{x}{2}\right). \tag{C.4}$$

Thus, we get the system of equations, taking (2.7) into account

$$\begin{cases} V_{ph} = \frac{\frac{n^2-1}{1-v^2} v\cos\theta + (1 + \frac{n^2-1}{1-v^2}(1 - v^2\cos^2\theta))^{\frac{1}{2}}}{1 + \frac{n^2-1}{1-v^2}}, \\ \cos\theta = \frac{1}{\beta V_{ph}}(V_{ph}^2 + (1 - V_{ph}^2)\frac{x}{2}). \end{cases}$$

Now the system takes the form

$$\begin{cases} V_{ph}(1 + y) = yv\cos\theta + (1 + y - yv^2\cos^2\theta)^{\frac{1}{2}}, \\ \cos\theta = \frac{1}{\beta V_{ph}}(V_{ph}^2 + (1 - V_{ph}^2)\frac{x}{2}). \end{cases}$$

Then we substitute the second equation into the first one

$$\begin{aligned} &V_{ph}(1 + y) - y\frac{v}{\beta V_{ph}}\left(V_{ph}^2\left(1 - \frac{x}{2}\right) + \frac{x}{2}\right) \\ &\quad = \left(1 + y - y\frac{v^2}{\beta^2 V_{ph}^2}\left(V_{ph}^2\left(1 - \frac{x}{2}\right) + \frac{x}{2}\right)^2\right)^{\frac{1}{2}}. \end{aligned} \tag{C.5}$$

This equation is equivalent to the system of equations

$$\begin{cases} V_{ph}^4(1 + y(1 - \frac{v}{\beta}(1 - \frac{x}{2}))^2) - V_{ph}^2(1 + xy\frac{v}{\beta}(1 - \frac{v}{\beta}(1 - \frac{x}{2}))) + \frac{1}{4}x^2 y\frac{v^2}{\beta^2} = 0, \\ V_{ph}^2 \geqslant \frac{xy\frac{v}{\beta}}{2(1 + y(1 - \frac{v}{\beta}(1 - \frac{x}{2})))}. \end{cases}$$

There we must note that following reasonings concern the case $v > 0$ (the particle and medium move in the same direction). We solve the first equation relative to the variable V_{ph}^2. Without taking the inequality for V_{ph}^2 into account we obtain two solutions

$$V_{ph}^2 = \frac{x^2 y \frac{v^2}{\beta^2}}{2(1 + xy\frac{v}{\beta}(1 - \frac{v}{\beta}(1 - \frac{x}{2})) \pm \sqrt{1 + 2xy\frac{v}{\beta}(1 - \frac{v}{\beta})})}. \tag{C.6}$$

Then we substitute these solutions in the inequality

$$\begin{aligned} &\frac{x^2 y \frac{v^2}{\beta^2}}{2(1 + xy\frac{v}{\beta}(1 - \frac{v}{\beta}(1 - \frac{x}{2})) \pm \sqrt{1 + 2xy\frac{v}{\beta}(1 - \frac{v}{\beta})})} \\ &\quad \geqslant \frac{xy\frac{v}{\beta}}{2(1 + y(1 - \frac{v}{\beta}(1 - \frac{x}{2})))}, \\ &\quad 1 - \frac{v}{\beta}x \pm \sqrt{1 + 2xy\frac{v}{\beta}\left(1 - \frac{v}{\beta}\right)} \leqslant 0. \end{aligned}$$

If one brings the considered inequality with the sign "−" it is deliberately valid for all θ. If one takes it with the sign "+" this becomes essential to do some work to prove that it is not valid for all θ. Let's write down the second equation of the system

$$x = \frac{2V_{ph}}{1 - V_{ph}^2}(\beta\cos\theta - V_{ph}),$$

from which it is obvious that Cherenkov photon is emitted if the projection of particle's speed on this direction is greater than phase speed of light in this direction. Let's assume that $x \geqslant 1$, then

$$\frac{2V_{ph}}{1 - V_{ph}^2}(\beta\cos\theta - V_{ph}) \geqslant 1,$$

which leads to

$$(V_{ph} - \beta\cos\theta)^2 + 1 - \beta^2\cos^2\theta \leqslant 0,$$

which is not right for all θ. Previous argumentation proves obvious physical statement that a particle cannot emit a photon with an energy greater than its own. We write the inequality allowing for $x < 1$

$$1 - \frac{v}{\beta}x + \sqrt{1 + 2xy\frac{v}{\beta}\left(1 - \frac{v}{\beta}\right)} \leqslant 0.$$

It is not right for all θ because its left part is less than 1. Thus, only one solution remains

$$V_{ph} = \frac{x\frac{v}{\beta}\sqrt{y}}{\sqrt{2(1 + xy\frac{v}{\beta}(1 - \frac{v}{\beta}(1 - \frac{x}{2})) - \sqrt{1 + 2xy\frac{v}{\beta}(1 - \frac{v}{\beta})})}}. \tag{C.7}$$

After rather boring calculations we get the expression for $\cos\theta$ in the case $v > 0$

$$\cos\theta_e = \frac{1 + \frac{\beta}{xyv}(1 - \sqrt{1 + 2xy\frac{v}{\beta}(1 - \frac{v}{\beta})})}{\sqrt{v^2 + \frac{2\beta^2}{x^2}(\frac{xv}{\beta}(1 - \frac{v}{\beta}) + \frac{1}{y}(1 - \sqrt{1 + 2xy\frac{v}{\beta}(1 - \frac{v}{\beta})}))}}. \tag{C.8}$$

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